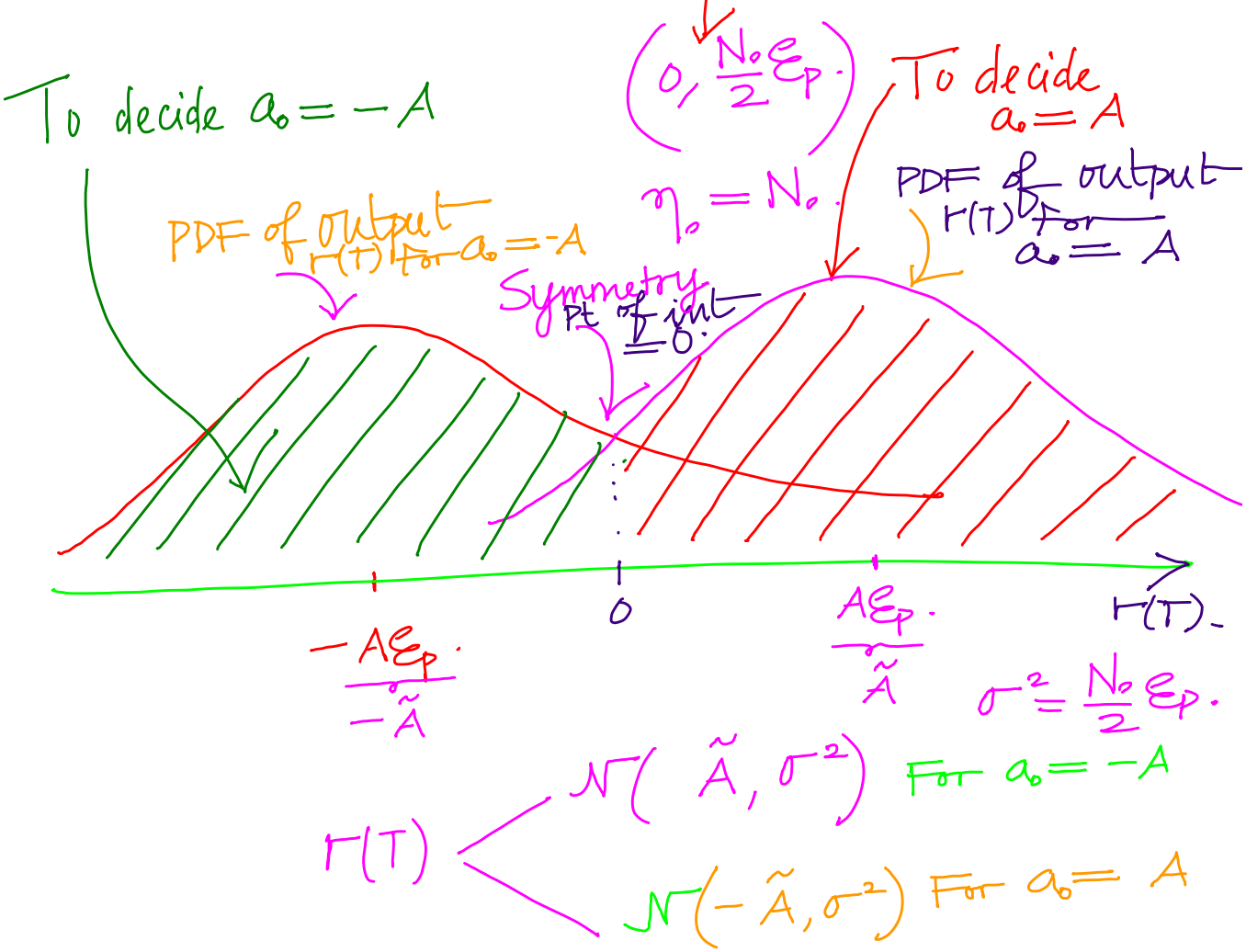
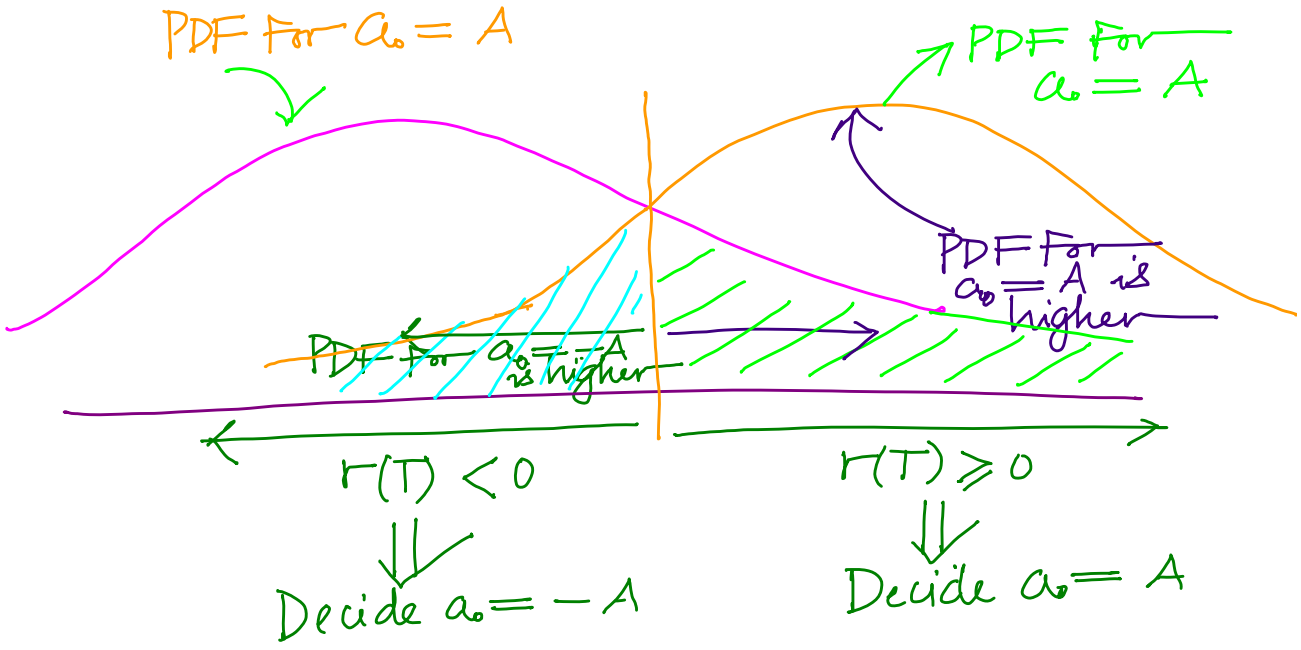
$$r(T) = \begin{cases} A\mathcal{E}_p + \tilde{n} \sim \mathcal{N}(A\mathcal{E}_p, \frac{N_0}{2}\mathcal{E}_p) \\ -A\mathcal{E}_p + \tilde{n} \sim \mathcal{N}(-A\mathcal{E}_p, \frac{N_0}{2}\mathcal{E}_p) \end{cases}$$





Decision Rule or Detection Rule.

$r(T) \geq 0 \Rightarrow$ Decide A

$r(T) < 0 \Rightarrow$ Decide $-A$

Corresponds to information bit 0

information bit = 1

Probability of Error

When does error occur?

Error occurs when $r(T) \geq 0$ for $a_0 = -A$ or when $r(T) < 0$ for $a_0 = A$

Consider the transmission of $a_0 = -A$

$$r(T) = -A\mathcal{E}_p + \tilde{n}$$
$$= -\tilde{A} + \tilde{n}$$

Error occurs if $r(T) \geq 0$

$$\Rightarrow -A\mathcal{E}_p + \tilde{n} \geq 0$$
$$\Rightarrow \boxed{\tilde{n} \geq A\mathcal{E}_p = \tilde{A}}$$

$$P_e = P_r(\tilde{n} \geq \tilde{A})$$

$f_N(\tilde{n}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{\tilde{n}^2}{2\sigma^2}}$

$$= \int_{\tilde{A}}^{\infty} f_N(\tilde{n})\, d\tilde{n}$$

$$= \int_{\tilde{A}}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{\tilde{n}^2}{2\sigma^2}} d\tilde{n}$$

$\frac{\tilde{n}}{\sigma} = n'$

$d\tilde{n} = \sigma\, dn'$

$$= \int_{\frac{\tilde{A}}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(n')^2}{2}} \cdot \sigma \, dn'$$

$$P_e = \int_{\frac{\tilde{A}}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(n')^2}{2}} dn' = Q\left(\frac{\tilde{A}}{\sigma}\right)$$

PDF of Standard Gaussian with mean = 0 var = 1

$$Q(u) = \int_{u}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

Gaussian Q Function Denotes the probability that the standard Gaussian RV is greater than u.

$$\boxed{P_e = Q\left(\frac{\tilde{A}}{\sigma}\right)}$$

Substitute values of $\tilde{A}, \sigma$

$$= Q\left(\frac{A\mathcal{E}_p}{\sqrt{\frac{N_0}{2}\mathcal{E}_p}}\right)$$

$$P_e = Q\left(\sqrt{\frac{A^2\mathcal{E}_p}{N_0/2}}\right)$$

Probability of Error

Gaussian Q-function